

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

Level - 2	Daily Tutorial Sheet 7
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86.(C) $2^x \left(1 + \frac{1}{2} + \frac{1}{4}\right) = 7^x \left(1 + \frac{1}{7} + \frac{1}{7^2}\right) \Rightarrow 2^x \left(\frac{7}{4}\right) = 7^x \left(\frac{57}{49}\right) \Rightarrow \left(\frac{7}{2}\right)^x = \left(\frac{343}{228}\right)$

$\Rightarrow$ Take log on both sides to get, or $(x) \log \left(\frac{7}{2}\right) = \log \left(\frac{343}{228}\right) \therefore x = \frac{\log \left(\frac{343}{228}\right)}{\log \left(\frac{7}{2}\right)} = \log_7 \left(\frac{343}{228}\right)$

87.(A) $(\log_x 2)(\log_{2x} 2) = \log_{4x} 2$

Here $x > 0$ and $x \neq 1, \frac{1}{2}, \frac{1}{4}$

$\Rightarrow \frac{1}{\log_2 x} \cdot \frac{1}{\log_2 2x} = \frac{1}{\log_2 4x}$ [Using : $\log_b a = \frac{1}{\log_a b}$]

$\Rightarrow \log_2 x \cdot (1 + \log_2 x) = (2 + \log_2 x)$ [Using : $\log ab = \log a + \log b$]

Put $\log_2 x = t$ to get, $t(1+t) = 2+t \Rightarrow t^2 = 2 \Rightarrow t = \pm\sqrt{2}$

Then $\log_2 x = \pm\sqrt{2}$

$x = 2^{\pm\sqrt{2}} \Rightarrow x = 2^{-\sqrt{2}}, 2^{\sqrt{2}}$

88.(C) $\left| \frac{(x-4)(x-1)}{(x-2)(x+2)} \right| \leq 1$

Case I : $x \in (-\infty, -2) \cup [1, 2) \cup [4, \infty)$

$\therefore \frac{x^2 - 5x + 4}{x^2 - 4} - 1 \leq 0 \Rightarrow \frac{5x - 8}{(x-2)(x+2)} \geq 0$

$x \in \left(-2, \frac{8}{5}\right] \cup [2, \infty)$

$\therefore x \in \left[1, \frac{8}{5}\right] \cup [4, \infty)$

Case II : $x \in (-2, 1) \cup (2, 4)$

$\therefore \frac{x^2 - 5x + 4}{x^2 - 4} + 1 \geq 0 \Rightarrow \frac{x(2x-5)}{(x-2)(x+2)} \geq 0$

$x \in (-\infty, -2) \cup [0, 2) \cup \left[\frac{5}{2}, \infty\right)$

$\therefore x \in [0, 1) \cup \left[\frac{5}{2}, 4\right)$

Complete solution $x \in \left[0, \frac{8}{5}\right] \cup \left[\frac{5}{2}, \infty\right)$

89.(C) $y = \frac{x^2 - 6x + 5}{x^2 + 2x + 1} \Rightarrow (1 - y)x^2 - (6 + 2y)x + (5 - y) = 0$

As $x \in R, D \geq 0 \Rightarrow D = (6 + 2y)^2 - 4(1 - y)(5 - y) \geq 0 \Rightarrow 12y + 4 \geq 0 \Rightarrow y \geq -\frac{1}{3}$

90.(B) Let $y = \frac{1 + x^2 - x}{1 + x^2 + x} \Rightarrow x^2(y - 1) + x(y + 1) + y - 1 = 0$

Discriminant of the above equation ≥ 0 (as $x \in R$)

$\Rightarrow (y + 1)^2 - 4(y - 1)^2 \geq 0 \Rightarrow -3y^2 + 10y - 3 \geq 0 \Rightarrow -(3y - 1)(y - 3) \geq 0 \Rightarrow (3y - 1)(y - 3) \leq 0$

$\Rightarrow 3\left(y - \frac{1}{3}\right)(y - 3) \leq 0$

91.(D) $\lambda(x^2 - x) + x + 5 = 0 \Rightarrow \lambda x^2 + (1 - \lambda)x + 5 = 0 \Rightarrow \alpha + \beta = \frac{-(1 - \lambda)}{\lambda} = \frac{\lambda - 1}{\lambda}, \alpha\beta = \frac{5}{\lambda}$

Now $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 4 \Rightarrow \alpha^2 + \beta^2 = 4\alpha\beta \Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 4\alpha\beta \Rightarrow (\alpha + \beta)^2 = 6\alpha\beta \Rightarrow \left(\frac{\lambda - 1}{\lambda}\right)^2 = 6\left(\frac{5}{\lambda}\right)$

$\Rightarrow \lambda^2 + 1 - 2\lambda - 30\lambda = 0 \Rightarrow \lambda^2 - 32\lambda + 1 = 0 \Rightarrow \lambda_1 + \lambda_2 = 32, \lambda_1\lambda_2 = 1$

$\Rightarrow \frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1} = \frac{\lambda_1^2 + \lambda_2^2}{\lambda_1\lambda_2} = \frac{(32)^2 - 2(1)}{1} = 1022$

92.(D) If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$

Then, $f(x)$ is minimum and $g(x)$ is maximum at $f(x) = \frac{-D}{4a}, \left(\because x = \frac{-b}{2a} \text{ and } f(x) = \frac{-D}{4a}\right)$

$\therefore \min\{f(x)\} = \frac{-(4b^2 - 8c^2)}{4} = (2c^2 - b^2) \text{ and } \max\{g(x)\} = -\frac{(4c^2 + 4b^2)}{4(-1)} = (b^2 + c^2)$

Since, $\min f(x) > \max g(x) \Rightarrow 2c^2 - b^2 > b^2 + c^2$

$c^2 > 2b^2 \Rightarrow |c| > \sqrt{2}|b|$

93.(A) Given, $\left|\frac{x-1}{x+2}\right| - 1 < 0$

Case 1 : When $x < 1, |x - 1| = 1 - x$

$\therefore \frac{1-x}{x+2} - 1 < 0 \Rightarrow \frac{-2x-1}{x+2} < 0 \Rightarrow \frac{2x+1}{x+2} > 0 \Rightarrow x < -2 \text{ or } x > -\frac{1}{2}$

But $x < 1$

$\therefore x \in (-\infty, -2) \cup \left(-\frac{1}{2}, 1\right) \dots \dots (i)$

Case 2 : When $x \geq 1, |x - 1| = x - 1$

$\therefore \frac{x-1}{x+2} - 1 < 0 \Rightarrow -\frac{3}{x+2} < 0 \Rightarrow \frac{3}{x+2} > 0 \Rightarrow x > -2$

But $x \geq 1$

$$\therefore x \geq 1, \text{ i.e., } x \in [1, \infty) \quad \dots \text{ (ii)}$$

$$\therefore \text{ From equations (i) and (ii), we get : } x \in (-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$$

94.(A) In general, a quadratic equation $ax^2 + bx + c = 0$ can have maximum 2-real roots. This happens when discriminant of the quadratic equation is non-negative i.e. $D \geq 0$.

In this question, it is given that equation $a(x+1)^2 + b(x^2 - 3x - 2) + x + 1 = 0$ is satisfied by all real values of x . This is possible only if given equation is an identity in x .

We know if $ax^2 + bx + c = 0$ is an identity in x , then $a = 0, b = 0, c = 0$.

Simplify the given equation and apply the result mentioned above, to get:

$$a(x^2 + 2x + 1) + b(x^2 - 3x - 2) + x + 1 = 0 \Rightarrow (a+b)x^2 + (2a-3b+1)x + a-2b+1 = 0$$

As it is an identity in x ,

$$a + b = 0 \quad \dots \text{ (i)}$$

$$2a - 3b + 1 = 0 \quad \dots \text{ (ii)}$$

$$a - 2b + 1 = 0 \quad \dots \text{ (iii)}$$

Solve (ii) and (iii) simultaneously to get: $a = 1$ and $b = 1 \Rightarrow (a, b) \equiv (1, 1)$

Replace (1, 1) in (i). (1, 1) does not satisfy (i). It shows there is no simultaneous solution (common solution) of the three equation (i), (ii) and (iii)

Hence there exists no pair (a, b) for which given equation is true for all $x \in R$.

95.(D) The given quadratic equation is satisfied by $x = -a, x = -b$ and $x = -c$.

In general, a quadratic equation can have maximum 2 roots. But given equation has 3 roots which is only possible if given equation is an identity and we know identity can have infinite roots.

$\Rightarrow$ Given equation has infinite roots.